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Which Is Greater: One Half or Two Fourths? An Examination of How Two Grade 1 Students Negotiate Meaning

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Abstract: A developing body of research in classroom mathematics discourse indicates that teacher facilitation can be specific and supportive without interfering in productive student talk. Students, including those who are struggling in mathematics or lack confidence, can benefit from exploring challenging mathematics and engaging in math-talk. In this study, one teacher videotaped two academically struggling Grade 1 students engaged in a mathematical debate. Transcript analysis illustrated that the student pair used math-talk guidelines to explain their thinking and ask one another clarifying questions, gaining a deeper understanding of equivalent fractions in the process.

Résumé: Un nombre croissant de recherches sur le discours mathématique en classe indiquent que l'intervention des enseignants peut être à la fois spécifique et favorable, sans pour autant entraver le discours productif des élèves. Les élèves, y compris ceux qui éprouvent des difficultés en mathématiques ou qui manquent de confiance en eux dans ce domaine, peuvent tirer avantage d'activités servant à explorer certaines idées mathématiques et à en discuter. Dans cette étude, un enseignant a filmé deux élèves de première année ayant des difficultés en mathématiques, au cours d'un débat sur une question mathématique. Une analyse des transcriptions montre que les deux élèves se sont servis des Directives sur le discours mathématique pour illustrer leur pensée et se poser l'un à l'autre des questions de clarification, ce qui leur a permis de mieux comprendre les équivalences de fractions.

BACKGROUND

Mathematics teachers and researchers may see the value in fostering mathematical discourse in the classroom, yet effective implementation of classroom math-talk is not a simple endeavour. Part of the challenge involves making decisions about how to deal with unanticipated and/or mathematically confusing ideas and about when to intervene and when to let a wrong answer stand so that students can think about ideas further. Much of the math-talk research focuses on whole-group discussions and consolidation stages of mathematics lessons (Ball, 1993; Bochicchio et al., 2009; Hufferd-Ackles, Fuson, & Gamoran-Sherin, 2004; Lampert, 2001; Smith, Hughes, Engle, & Stein, 2009; Staples, 2007; Staples & Colonis, 2007; Stein, Engle, Smith & Hughes, 2008), rather than on the role and activity of the small-group or pairs discussion. Math-talk by

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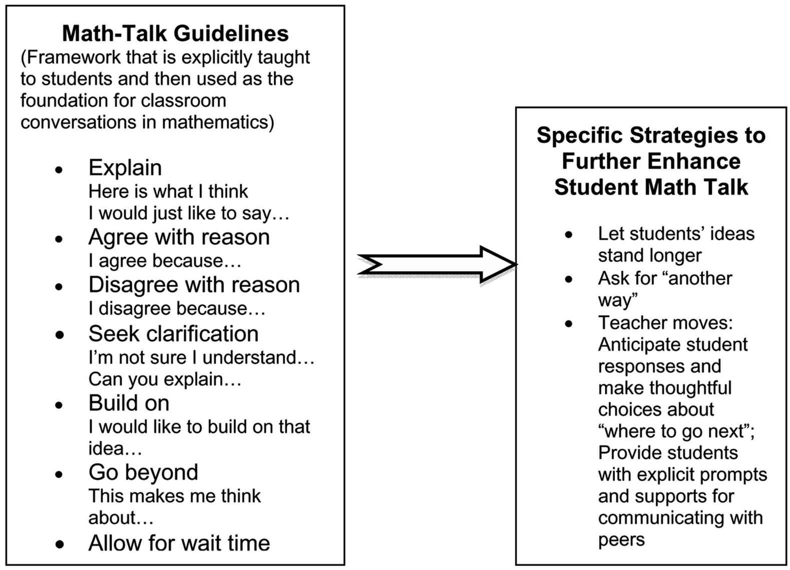

FIGURE 1 Research-Generated Math-Talk Guidelines.

students in pairs is an important feature of student communication in mathematics classrooms, but it seems to be harder to facilitate and assess (Civil, 1998; King, 2002).

In an Ontario school, a teacher implemented math-talk guidelines (see Figure 1) with her Grade 1 class as part of a mixed methods research study that focused on mathematics communication using a lesson study (Lewis, Perry, & Murata, 2006) professional learning model. The math-talk guidelines were published in a 2007 research monograph and identify specific strategies for enhancing student communication (Bruce, 2007). Having established the norms for math-talk in her room, this teacher began to consider more sophisticated structures to support the students in their communication of mathematics. One of her strategies was to let student ideas that were mathematically unsound or confusing to stand longer (VanChloest & Envart, 1998). This study examines the discussion between a pair of academically challenged Grade 1 students exploring fractions concepts. The extended discussion between the two students was videotaped by the teacher, who was interested in collecting evidence of student–student dialogue, with the support of researchers who were collectively engaged in lesson study activity. Video and transcript evidence illustrated how the students engaged in a sustained discussion about equivalent fractions. The structures and scaffolding that the teacher implemented were tailored to student needs that the teacher had identified. The sustained discussion revealed flaws or misconceptions, in the more confident student's (Alex's) thinking. As the discussion continued, the structure provided by the teacher allowed Chloe, the other student, to articulate her ideas with increasing confidence and eventually affected Alex's emerging understanding of part–whole relationships. Our analysis of the discussion between these two Grade 1 students reveals the importance of sustained math-talk that enables students to present their ideas in a supportive environment and to question one another in productive ways.

THEORETICAL FRAMING

Developing a Math-Talk Learning Community

Walshaw and Anthony (2008) provided a comprehensive review of the literature on mathematical discourse with an emphasis on revealing what teachers actually do to manage classroom discourse. They identified pedagogical approaches to classroom discourse that produced desirable outcomes for diverse students. Their meta-analysis provides strong evidence that student math-talk is a nonnegotiable component of effective instructional practice. The authors are clear that communication in mathematics classrooms does not automatically translate to enhanced student understanding but that "better understanding is dependent on particular pedagogical approaches, purposefully focused on developing a discourse culture that elicits clarification and produces consensus within the classroom community" (p. 522). The pedagogical approaches identified include: developing a community of learners where participation carries rights and obligations; careful listening, synthesizing, and questioning of student responses to move mathematical thinking forward; sharing and modeling conventional mathematical language; and providing regular opportunities for students to engage in mathematical argumentation.

Two of these named pedagogical strategies apply directly to the teaching episode on which our study is based: provision of opportunities for students to explore and construct mathematical arguments and the establishment of a learning community. The establishment of a community of learners—where students feel safe volunteering ideas and taking risks—is a necessary precondition to effective communication in the problem-solving classroom. As Walshaw and Anthony (2008) pointed out, "teaching for inclusion ensures that participation in classroom discussion is safe for all students" (p. 527).

Though students need responsive (as opposed to directive) pedagogical support (Manouchehri & Enderson, 1999), overscaffolding may actually be counterproductive to learning (Walshaw & Anthony, 2008). Such support is often well intentioned, in the interest of helping students avoid mistakes and misconceptions. Yet mistakes are an important part of mathematical learning, what Borasi (1994) called "springboards for inquiry" (p. 169). Eggleton and Moldavan (2001) asserted that mistakes are an inevitable part of problem solving and, indeed, that "if no mistakes are made, then almost certainly no problem solving is taking place" (p. 44). These authors prescribe that teachers should allow students to work through a problem with minimal intervention, even when erroneous thinking is taking place. Teacher–researcher Ann Enyart shared the challenges of exploring this pedagogical approach in a 1998 study (Van Chloest & Enyart, 1998). Enyart participated in a research study in which teachers were asked to compare videotapes of themselves teaching before and after reading the National Council of Teachers of Mathematics' (NCTM, 1991) standards for discourse. The teachers were asked to reflect on the change in their teaching. Enyart's firsthand account is a fascinating read of her learning about the importance of communication in mathematics and about strategies for fostering this dialogue, including letting wrong answers stand as students explore misconceptions:

> One pleasant surprise was finding that if I let a wrong answer stand, it usually did not last very long without some comment. . . . Allowing a student to leave the classroom with a mathematically incorrect idea is not as horrifying as I had once thought. First, if a student has a misconception, it can be so deeply rooted that he or she may not pay attention to the teacher's correction of the process.

> Second, giving the student a day to think about it does not mean that she or he will embrace the misconception more tightly. (p. 154)

Of course, knowing when to step in and intervene is part of critical teacher knowledge that is essential for moving student thinking forward (Walshaw & Anthony, 2008).

Further to creating a community and culture of care and respect in the classroom, Yackel and Cobb (1996) reported that in order to extend student thinking through debate, teachers must establish norms for what productive mathematical debate looks and sounds like, teaching students what makes an effective mathematical explanation. In 2007, Bruce reported effects of the use of math-talk guidelines (see Figure 1) as a framework for mathematical dialogue in classrooms. The guidelines were field tested and then introduced in a district-wide focus on math-talk with Grade 6 teachers. The implementation of the math-talk guidelines by Grade 6 teachers facilitated increased discussion in the mathematics classroom among teachers and students as well as increased student achievement in a quasi-experimental design (Ross & Bruce, 2007). This study demonstrates that by introducing and then shoring up the guidelines structure, even very young mathematicians are able to carry out mathematically productive discussions and debates.

Mathematics Communication of Struggling Learners

The reform movement in mathematics has experienced mixed results for low-achieving students (Baxter, Woodward, & Olson, 2001; Geary, 1994; Kroesbergen, Van Luit, & Maas, 2004; Lubienski, 2000; Ross, Xu, & Ford, 2008; Woodward & Brown, 2006), where some studies have found direct instruction and explicit guidelines to be more helpful to struggling students than open-ended problem-solving approaches.

On the other hand, the use of mathematical models and manipulatives are also proving to have a positive effect (see Woodward & Brown, 2006) on low-achieving mathematics students, as is explicit attention to developing positive student beliefs about their abilities to learn mathematics (Bandura, 1986; Ross, Hogaboam-Gray, & Rolheiser, 2002; Shunk & Pajares, 2001). However, one of the main issues for low-achieving mathematics students seems to be related to the challenges of communicating mathematics ideas in whole class discussions and in small group and pairs activities. Nonetheless, teacher use of specific structures such as open discussion of problems encountered during mathematics tasks, comparing multiple solutions and solution strategies, and highlighting and resolving opposing perspectives and mathematical claims is leading to greater student understanding for not only high- and mid-achieving students but also for low-achieving students (Empson, 2003). In a close analysis of two low-achieving students working with fractions, Empson found that there were three factors that contributed to the pair's success: (a) the use of tasks that elicited the students' prior understanding and experiences (building legitimacy of student thinking and ownership of ideas); (b) the use of frameworks/norms/guidelines that treated the students as mathematically competent (where students were animated problem solvers, claim makers, and solution reporters); and (c) frequent opportunities for interactions (reinforcing intellectual practices that are scaffolded). Student–student communication, though in focus in Empson's study, was secondary to teacher moves that supported small-group and whole-class communication of mathematics ideas.

Student–student communication in mathematics for low-achieving students is an area of research that is particularly limited. This article contributes to the literature on student–student

communication with an emphasis on student math-talk where there is *minimal* teacher redirection, illustrating how one pair of low-achieving students was able to construct meaning together, with the support of the same three factors identified in the Empson (2003) study: The pair of Grade 1 students in this study had access to (a) a mathematics task that was challenging but built on prior student experience; (b) the use of math-talk guidelines and related prompt "popsicle sticks" as well as physical models (paper fraction pieces) provided by the teacher that treated the students as problem solvers and claim makers; and (c) sustained opportunities to continue the discussion (over several lessons on equivalent fractions), with limited teacher redirection.

CONTEXT

The study reported here is a part of ongoing research on lesson study in mathematics classrooms, where teams of teachers engage in lesson study as a form of professional learning. Maggie is one of the participants in the lesson study project and teaches Grade 1 at a small school (220 students) in a mid-sized town in central Ontario. The lesson study team at Maggie's school decided to focus on student mathematics communication. As part of that study, Maggie designed an eight-lesson sequence to explore early fractions concepts. The sequence moved through problem-based tasks with multiple solution strategies, exploring the benchmark fractions of 1/2 and 1/4. For example, on day 2, the students were to divide a set of cubes in half, but the teacher gave the students an odd number of cubes to work with. On day 3, the students were given a problem where two different sized pizzas were divided in half, to explore notions of same sized area objects/models when making comparisons (see Table 1 for an overview of the tasks in the lesson sequence).

On the seventh day, Maggie told the class that they were going to work in partners to talk about a fraction problem, at the end of which they had to reach agreement as a pair. She was explicit that both students would have to explain their thinking and be ready to make arguments to prove their points. Maggie gave each student two popsicle sticks—one that had "I agree because . . ." written on it, and one that said "I disagree because . . ."—and told the students that they had to lay down at least one popsicle stick during the conversation. The teacher modeled this and then gave the students the problem to discuss: Which is bigger? Two quarters or one half? Students were also given a paper square that had lines on it to illustrate four quarters (see Figure 2). The students were instructed to cut the larger square into four and use these pieces to help them figure out their solution to the question. This flexible area model proved to be an important reference point for the students as they talked through their ideas with their partner and used the pieces of the square to try to convince one another of their views.

Maggie videotaped two focus students that she was interested in learning about in more detail. Later, the teacher also conducted whole-class discussions to further clarify ideas and guide mathematics discussion. Maggie was interested in this pair in particular because of her observations of these students in math class. She noticed that Alex was a very confident student in mathematics (whose confidence sometimes got in the way of listening to other ideas and who tended to be dominant in pair discussion) but mathematically was struggling. Chloe was experiencing difficulty in mathematics as well but, unlike Alex, was tentative about her thinking and tended to be quieter, less confident, and less persistent in pair and whole-class discussions. Maggie shared her observations with the researchers and decided to videotape them to gain additional insights into the ways that they communicated with one another to develop their

TABLE 1
Outline of Math Lesson Sequence in Fractions Used by Maggie

Day and focus	Lesson activity and general observations
Lesson 1: Introduction to one half (parts of a whole: two equal portions)	Teacher gave each student a square paper and asked them to fold it in half. Students talked about how they did this and how they knew it was folded in half. Students explored various shapes on interactive whiteboard with partitioning lines. Some had lines dividing the shape in half. Others had lines that partitioned the shapes unevenly. Students also drew lines on shapes to illustrate their thinking. Class discussion.
Lesson 2: Introduction to one half (parts of a set)	Teacher put a pile of cubes in the center of the carpet. Teacher told students that she was going to put half the cubes "here" and the other half "there." Teacher purposely put more than half in one pile but made it difficult to tell (and used an odd number of cubes). Teacher asked if she had divided the pile in half. Some students said yes. Others said no. Class discussion. Students had to figure out what they could do to divide an odd number of cubes evenly.
Lesson 3: Introduction to one quarter, problem solving about halves (comparable areas)	Students explored shading quarters to make halves using area models. Class discussion. Partner problem solving: Danni was hungry and ate half his family's pizza. Susan was hungry, too, and ate half of her family's pizza. But Danni ate more than Susan. How can that be?
Lesson 4: Introduction to quarters	More discussions and use of models to explore 1 quarter, 2 quarters, 3 quarters, and a whole. Also discussed the term *fraction*.
Lesson 5: Quarters representations in area models	Area models on interactive whiteboard: some quartered with a vertical and horizontal line, some quartered with diagonal lines. Class discussion about what would represent one quarter on each of these area models. Then students were asked to compare the quarters to determine whether they represented the same amounts.
Lesson 6: Equivalent quarters even when area model is not the exact same shape	Students given area models in paper. They had to cut out one quarter from shape A and then make it fit perfectly into shape B's quarter (involved cutting and pasting).
Lesson 7: Comparing two quarters and one half (video episodes)	Student pairs activity using math-talk guidelines: Introduced the popsicle sticks (one that says "I agree because . . ." and another that says "I disagree because . . ."). Teacher explained that students had to lay down at least one popsicle stick during the conversation. Teacher modeled and then gave students a question: What's bigger? One half or two quarters? Or are they the same? Students were also given a paper square that had lines on it (one vertical line, and one horizontal line).
Lesson 8: Reinforcing the relationship between two quarters and one half	Whole-group discussion about the task of lesson 7 using the interactive whiteboard. Teacher had two identical squares divided into quarters and engaged students in shading and labeling halves and quarters to build consensus, as well as layering the squares on top of one another on the interactive whiteboard to make comparisons. The students had paper models as well as those provided visually on the interactive whiteboard.

mathematics understanding. The task set out for students appeared to be simple but led to rich discussion for these Grade 1 students. During the paired discussion, there was minimal intervention by the teacher (she asked only one question over the course of 9 minutes and 41 seconds of analyzed transcript).

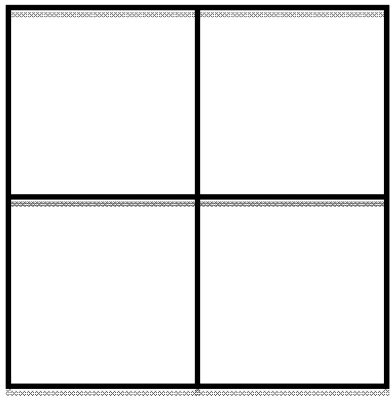

FIGURE 2 The Square Region, Divided Into Quarters.

METHOD

Participants

Maggie, our Grade 1 teacher, engaged in 3 years of lesson study research at her school and with the researcher authors of this article. Together, we frequently discussed challenges and successes of the mathematics communication of students in her classroom. Researchers were in Maggie's classroom observing throughout the study, but Maggie also took an interest in observing her students through the use of video. One teacher move that Maggie was interested in, and discussed with researchers, to support student–student communication and meaning making was to let student ideas stand longer and to treat their ideas as legitimate and important. She was not convinced that 6- and 7-year-olds would be able to sustain a mathematics conversation. Maggie also explained that she found this hard to do at first, because if a student had a wrong idea, she wanted to correct it immediately. She found however, that when students struggled with the idea longer, they revealed misconceptions that she could then address, and it built deeper understanding of the concept (field notes from team meetings). The students Maggie selected to videotape, Chloe and Alex, were both struggling academic students, as assessed by PRIME diagnostics (Small, 2005) and teachers' observations. Chloe and Alex were willing to share ideas with one another, but neither had a solid understanding of fractions.

Data Sources

Although researchers gathered extensive video footage over the course of the research study, teachers gathered additional video footage on a more regular basis in their classrooms. In Maggie's case, she brought the video footage to the attention of the researchers and asked for feedback and additional possible interpretations of the interaction. Maggie was pleased with the sustained discussion of the two students and with her restraint in not interrupting with the student–student math-talk. The researchers transcribed the entire series of conversations between the two students and conducted an analysis based on the unit of an utterance (Rowe, 2004), defined as "an

uninterrupted stretch of speaking" (p. 88). The transcript with analysis annotations was then returned to Maggie for her reaction. Three particular moments in the full transcript caused the most discussion and interest on the part of Maggie and the researchers because they represented the key stages of the discussion between the two students: (a) The students made a first attempt at thinking about the problem. (b) The students were confused about mathematics terminology, such as what the term *quarter* means, which in turn confused their understanding of the problem. (c) The students engaged in a series of questions and responses to begin to clarify and consolidate their understanding of halves and quarters. These three segments were considered important stages in the students' construction of fractions concepts related to equivalent fractions. A fourth point of discussion between Maggie and the researchers involved reevaluating the use of paper cut-out squares where the quarters were fully separated from the whole.

Framework for Analysis

In the micro-analysis of the three transcript segments that follow, we do not attempt to use the fine-grained lens of discourse analysis (see Radford 2000a, 2000b, 2003, 2005, for an example of precise gesture and discourse analysis). Our goal in analysis was to examine how the two students were building their mathematics understanding of equivalent fractions through the *types* of interactions they engaged in when using the math-talk guidelines (Bruce, 2007). We were unable to find an analysis tool for this purpose and therefore decided to apply the use of the whole-class analysis tool developed by Hufferd-Ackles et al. (2004). In summary, Hufferd-Ackles et al. were able to develop descriptions of math-talk communities based on case study observations in classrooms over a one-year period. They describe a continuum of types of math-talk for both teachers and students, which moves from teacher-directed communication increasingly toward more student-centered communication where students are responsible for the ideas and discussion in the room, while being supported by the teacher. For this article, we analyzed the peer communication between Alex and Chloe, using the student descriptors at the student-directed end of the continuum exclusively. We evaluated the application of the whole-class interaction analysis tool to a paired student interaction. To this end, we were guided by the four distinct but related types of math-talk identified by Hufferd-Ackles et al. (a) questioning; (b) explaining mathematical thinking; (c) source of mathematics ideas; and (d) responsibility for learning.

Questioning

The math-talk continuum includes the following description as high-quality questioning:

> Student-to-student talk is student initiated, not dependent on the teacher. Students ask questions and listen to responses. Many questions are "Why?" questions that require justification from the person answering. Students repeat their own or other's questions until satisfied with answers. (Hufferd-Ackles et al., 2004, p. 90)

In cases where Chloe or Alex asked their partner a question, we coded this as "questioning" (QU).

Explaining Mathematical Thinking

When either student gave a relatively complete explanation, we coded this for "explaining mathematical thinking" (EXP). Explaining mathematical thinking at the highest level is described by Hufferd-Ackles et al. as:

> Students describe more complete strategies: they defend and justify their answers with little prompting from the teacher. Students realize that they will be asked questions from other students when they finish, so they are motivated and careful to be thorough. Others students support with active listening. (2004, p. 90)

Responsibility

The math-talk continuum describes high-quality "responsibility for learning" as:

> Students listen to understand, then initiate clarifying of other students' work and ideas for themselves and for others during whole-class discussions as well as in small group and pair work. Students assist each other in understanding and correcting errors. (Hufferd-Ackles et al., 2004, p. 90)

We interpreted responsibility for learning in the context of one pair of Grade 1 students to mean a display of owning the learning in one of three ways: (a) by either agreeing or (b) disagreeing with one another explicitly, preferably with reason (we have coded this as RESP-A or RESP-D) or (c) by seeking points of clarification (RESP-C).

Source of Mathematical Ideas

Finally, "source of mathematical ideas" refers to who is initiating or prompting math-talk. Hufferd-Ackles et al. (2004) described highly developed math-talk where: "Students spontaneously compare and contrast and build on ideas. Student ideas form part of the content of many math lessons" (p. 90). In our case, the teacher posed the initial question to students; however, the pairs were then left on their own to discuss their thinking. The source of mathematical ideas rested squarely on the students; therefore, it was not particularly helpful to code the segments for this characteristic.

ANALYSIS AND RESULTS

In the three following segments, we see Chloe and Alex attempting to understand equivalent fractions but they are struggling with the part–whole relationship and the language of quarters, halves, and wholes. Rather than intervening quickly, the teacher allows the students to discuss ideas in their pair and, in so doing, reveal their misconceptions and struggles. Each of the segments is part of a continuation of the same conversation. They are approximately the same length and reflect three points in an extended conversation between the two students: Alex and Chloe. The formatting of the transcript analysis was inspired by Edward Tufte's (1990) presentation of David Hellerstein's article, "The Slow Costly Death of Mrs. K__," originally published in *Harper's* in 1984. Tufte illustrated Hellerstein's annotations in wide margins surrounding the text in order to describe the details of Mrs. K's hospital costs, generating an unusual narrative through the

analysis of Mrs. K's hospital bill. Tufte referred to this as "visually stratifying various aspects of the data" in order to reduce noise and clutter (p. 53).

Prior to the discussion between Chloe and Alex, large squares were cut into four small squares (quarters). The pair of students placed the squares on the table in front of them, along with their communication popsicle sticks that state: "I agree because . . ." and "I disagree because . . ." The prompt on the interactive whiteboard is "Which is bigger? Two quarters or one half?"

In this first segment, both Alex and Chloe explain their thinking. Alex appears confident, speaking firmly, but uses incorrect mathematics terminology (such as "quarter" to mean the large square made up of quarters) and *may* be relying on the denominator rather than considering the relationship between the numerator and the denominator. Chloe wants to agree with her confident partner but hesitates. She asks clarifying questions to try to understand Alex's position. The students do not seek help from their teacher during the discussion. The coding table (Table 2b) summarizes the characteristics of the interactions in segment one.

As the discussion continues, the two students persist in trying to explain their understanding to one another.

By the end of segment 2, Alex and Chloe are farther apart in their understanding. Chloe has convinced herself that one half and two quarters are equal, whereas Alex has decided that two quarters can be represented by two large squares each partitioned into 4 smaller squares. In segment 2, Chloe disagrees with Alex more openly and seeks clarification from Alex consistently. The coding table (Table 3b) summarizes the characteristics of the interactions in segment two.

Later, Chloe was asked by the teacher to present her mathematical ideas to Alex and the teacher using the paper squares and an interactive whiteboard.

In segment 3, the teacher asked Chloe to explain her thinking to Alex further. This gives Chloe's ideas legitimacy, and she uses several different strategies for explaining how one half and two quarters are equal: She uses the paper squares, then she uses a subtraction strategy, then she uses diagrams on the interactive whiteboard, and, finally, she tries to give an active example of the teacher showing one half and two quarters. Throughout these explanations, Chloe's mathematics vocabulary increases in precision. At the same time, Alex asks for clarification six times, a reversal of their prior roles in the pairing. Although the teacher has only interjected on one occasion during the three segments, it appears that her request for Chloe to explain her thinking further gave Chloe the confidence to persist and improve her explanations. It also seemed to have caused Alex to listen to her partner more closely, in an effort to truly understand what Chloe was trying to tell her. The coding table (Table 4b) summarizes the characteristics of the interactions in segment 3.

In analyzing the three text segments using the Hufferd-Ackles et al. (2004) characteristics of math-talk, we can summarize the interaction with a totals count table (see Table 5).

It is important to remember that these two students were 6 years of age, in Grade 1, yet they carried on lengthy conversations (of which we have only provided excerpts that reflect the essence of the entire conversation sequence) where they persist in explaining their viewpoints and attempt to use mathematics models to support their explanations. There were 15 examples of the students explaining their mathematics thinking and 13 examples of the students seeking clarification from one another across the three segments. Although initially they wanted to agree with one another, the students later disagreed formally on seven occasions. The popsicle sticks ensured that students had a physical tool to signify agreement or disagreement and to help them assert their opinions. This validated student opinions and provided structure to the discussion, especially important for giving voice to the more tentative student in the pair.

TABLE 2a
Segment 1: Initial Exploration (2 minutes, 28 seconds excerpt)

Chloe		Alex
Chloe is explaining her thinking: Seems to be initial agreement, but the students carry on to disagree. (EXP)	Chloe: This is a half [putting two squares down] but this is two quarters [putting the remaining two squares down beside them].	
	Alex: So this is a whole half and this is 2 quarters . . . what did you say?	Alex seeks further clarification in the form of a question. (QU) (RESP-C)
	Chloe: This is a whole half and this is two quarters.	
	Alex: But they're the exact same [confused look].	Alex doesn't think it makes sense for them to be equal. (RESP-D)
Chloe agrees that they are the same, but then she thinks that one half may be bigger (and is perhaps comparing one half to one quarter or is losing track of the anchor of one whole). (QU)	Chloe: [. . . thinking . . .] Yeah they are the exact same . . . okay, I agree [tentative, reaching for and putting down the "I agree" popsicle stick], but isn't half bigger, because it's like, half of this whole square?	
	Alex: But half, between this [points to 3 squares] and one more [adding one quarter to complete the square] would be bigger than just that [indicating the one small square].	Alex recreates the square and compares the whole large square to one quarter (small square) moving away from the question. (EXP)
Chloe uses popsicle stick to agree with Alex but asks a follow-up question. (RESP-A) (QU) (RESP-C)	Chloe: Half . . . oh yeah, it would be bigger, I agree [places the popsicle stick down] . . . yeah, but if half is . . . isn't this half though [indicating 2 of 4 squares] . . . and this is, um [starting to reach for other squares].	
	Alex: A quarter is this [arranging 3 small squares and then refers to and adds the fourth small square to make a larger square] . . . that would be just like a fourth [points to large composite square] . . . and that is half [indicating 2 of the small squares].	Alex is really struggling to explain her understanding with words. She seems to have confused "one quarter" and "four quarters." (EXP)
Chloe uses limited language combined with manipulation of the squares to explain how two fourths is the same as one half. (EXP)	Chloe: But . . . you take 2 of a quarter, so . . . [moving 2 of 4 squares away].	
	Alex: Actually a quarter is just like fourths and fourths is just like that [arranging four squares back into a larger square] and half is just 2, so if you were comparing that [indicating 2 squares] to this [four squares arranged into a larger square], this would be bigger than that . . . because there's two more extra ones in the quarter, and that one only has two.	Alex is referring to the whole (4/4) when she says quarter. She states that fourths and quarter are the same and points to the large square made up of four small squares. Then she names $\frac{1}{2}$ to be 2 of the four small squares. This is an example of a common misconception of seeing the parts as a counting operation rather than parts of a larger whole. (EXP)

(Continued on next page)

TABLE 2a
Segment 1: Initial Exploration (2 minutes, 28 seconds excerpt) *(Continued)*

Chloe		Alex
Chloe is trying to point to a flaw in Alex's language: If you have two quarters, then according to Alex's definition, you would have two large squares; Chloe returns to the question and the use of the term "two quarters," then asks another question for clarification. Alex's use of "quarter" to mean the whole (4/4) is confusing to Chloe. (QU) (RESP-C)	Chloe: . . . but doesn't . . . the question say you have to have two quarters . . . so this would be two of a whole quarter?	

Even though Maggie, the teacher, only intervened one time, this became a turning point in the conversation. Chloe felt validated in being asked to explain her understanding and proceeded with confidence. Alex was able to ask her partner for additional clarification to help her better understand Chloe's perspective. Although not part of the segments shared in this article, Alex was able to eventually understand the relationship between the two fractions and, in particular, to clarify the significance of both the numerator and the denominator. The teacher facilitated the exploration of mathematical ideas with math-talk guidelines and structures; however, she did not interfere with the pair's discussions.

DISCUSSSION

Reflection on a Conversation With Maggie

One area of discussion among the teacher and the researchers during the study was the role of the regional model and the effects of cutting the large square into quarters at the onset of the task

TABLE 2b
Summary Analysis of Types of Interaction in Segment 1

	Chloe	Alex	Total
Questioning (QU)	3	0	3
Explaining mathematical thinking (EXP)	2	3	5
Responsibility for learning (RESP)			
Agree (RESP-A)	1	0	1
Disagree (RESP-D)	0	1	1
Seeking clarification (RESP-C)	2	1	3

TABLE 3a
Segment 2: Further Debate (3 minutes, 53 seconds excerpt)

Chloe		Alex
Chloe uses the disagree (RESP-D) stick and asks a clarifying question that is directly tied to the task. (RESP-C)	Chloe: [pauses to look at the popsicle sticks, choosing one] I don't agree, because . . . well, the quarter . . . she said in the question, which is bigger, two quarters or one half?	
	Alex: But the quarter is bigger than the half	Alex disagrees without using the scaffolding of the popsicle stick. (RESP-D)
Chloe wants to agree with Alex again, but instead she asks a clarifying question. (QU) (RESP-C)	Chloe: Okay, [looking at her popsicle stick] I a . . . [she begins to say "agree"] . . . but, which is bigger?	
	Alex: This is because this is the fourths and this is only two and this is four, so four is bigger than two, so that one must be bigger.	For Alex, the number 4 is greater than the number 2. She is not clear on the part–whole relationship—this confirms Alex's earlier misconception. (EXP)
Chloe begins to explain but doesn't know how to finish. (RESP-C)	Chloe: But . . . okay, it says take two pieces of the half [pulling two small squares away from the four square].	
	Alex: [thinking . . .] but this is a quarter [points to the four small squares], that's the half [two squares].	Alex is calling the four small squares that make one large whole "one quarter" and the two small squares "one half." One interpretation is that the denominator is all that matters for Alex. (EXP)
Chloe finally accesses the language for two quarters demonstrating an increase in the precision of the language. (EXP). She also refers to the disagree stick. (RESP-D)	Chloe: Two pieces of the quarter [puts her hand on two of the four small squares and then referring back to the "I disagree" popsicle stick] . . . well it's supposed to be two quarters [gestures at the two small squares].	
	Alex: If it said two quarters it would be like that [puts 8 squares together], that's two quarters.	Alex explains that eight small squares represent what she calls "two quarters." The whole is a shifting target. (EXP)
Chloe redirects Alex to the teacher question. (RESP-C)	Chloe reads the question again [to Alex]: "Which is bigger, two quarters or one half?"	
	Alex: two quarters, see? . . .	Alex persists in her misconception that one quarter refers to the whole square. (EXP)
Chloe grasps onto and holds this thinking of $\frac{1}{2}$ and 2/4 as being equal. (RESP-D) [Note that this is the first time Chloe is not using the scaffolding of the popsicle stick]	Chloe: so two would be . . . it's equal. It's equal.	

TABLE 3b
Summary Analysis of Types of Interaction in Segment 2

	Chloe	Alex	Total
Questioning (QU)	1	0	1
Explaining mathematical thinking (EXP)	1	4	5
Responsibility for learning (RESP)			
Agree (RESP-A)	0	0	0
Disagree (RESP-D)	3	1	4
Seeking clarification (RESP-C)	4	0	4

rather than using some other strategy such as folding lines where the large square was folded into quarters and then unfolded. Maggie suspected that the issue of a "moving target" where the small squares (regions of the area model) were no longer literally connected to the large square (the area model in whole) may have contributed to confusion for Alex in particular. Gould, Outhred, and Mitchelmore (2006) underline that "pre-partitioned shapes may encourage a double count rather than an area-based interpretation of a regional model" (p. 262). In closely observing and listening to these students, it is apparent that not only can we learn about student misconceptions and understanding, but we can also learn about task design and more effective representations of mathematical ideas to support student understanding.

Use of the Hufferd-Ackles Math-Talk Framework as an Analysis Tool for Pairs Interaction

In applying the Hufferd-Ackles et al. (2004) analysis tool for whole group math-talk to a pair's discussion, we found that all four of the characteristics were indeed present. One of these, source of mathematical ideas, was not coded because there were no dynamic conversations between the teacher and the students. Although the teacher prompted the discussion, the students in these segments were the only source of mathematical ideas. Coding based on the remaining three characteristics allowed us to do two important things: (a) tease apart the *types* of interaction occurring between Alex and Chloe; and (b) pinpoint how the roles of the students and the conversation dynamics changed over time (by counting the number of times each type of interaction was used by each student in each segment). The summary tables illustrate that both students had frequent opportunities to explain their mathematical thinking and to seek clarification from one another but that each student's use of the three types of interaction changed as the discussion continued (see Table 5). Interestingly, the number of explanations given by either student was not connected to the accuracy of their mathematical thinking but rather to confidence. That is, even though Alex's understanding of equivalent fractions was weak, she had the confidence to explain her ideas, dominating this type of interaction in the first two excerpts. Once Chloe received validation from the teacher that her ideas were worth explaining, Chloe gained the confidence to further justify her thinking using precise mathematical language, dominating this type of interaction in the final excerpt. Further, as one student was explaining her ideas, the other student asked for clarification of those ideas as indicated in the summary tables, suggesting that these

TABLE 4a
Segment 3: Chloe Tries Other Ways to Explain Her Thinking After the Teacher Prompts Her to Explain
(3 minutes, 20 seconds excerpt)

Chloe		Alex
Chloe explains that one half and two fourths are the same but still struggles with some of the fractions language. (EXP)	Chloe: Sure, um, one half is equal [to two fourths] because this is one half [points to 2 small squares] and this is two quarters [points to 2 small squares]. Because if we have a half, and we divide it into two quarters, it's half of the whole, so it comes together and it makes the whole square . . . There's always two pieces equal in the whole, because it's just like the half. [Alex is looking skeptical.] It's true!	Alex's posture indicates that she is listening closely.
	Alex: Wait, can you say that again?	Alex asks for clarification. (QU) (RESP-C)
Chloe uses the small squares again to show her thinking and uses more precise language. (EXP)	Chloe: Two quarters is the same [as one half], because this is one whole half [shows two small squares] and this is two quarters [shows the other two small squares], so it's the same because the half is exactly as big as the two quarters.	
	Alex: How is it bigger?	Alex thinks that Chloe is trying to explain that one is bigger than the other. (QU) (RESP-C)
Chloe clarifies that they are equal and illustrates how all the small squares are of equal size. She also attempts a subtraction strategy (which is unhelpful to Alex). (EXP) (RESP-D)	Chloe: No, it's not bigger, it's equal. They're the same. Because each square [pointing to each half of the larger square] is divided into two pieces and they're not bigger, see. And if you add one half to the other two, you get quarters. So if you take away, it has to be half, and then it's equal.	
	Alex: I don't get what you're saying.	Alex states that she is confused by Chloe's explanation. (RESP-C)
Chloe changes explanation strategies and shows her thinking on the interactive whiteboard with a diagram to clarify the ideas. (EXP)	Chloe: Look up there, Alex [pointing to the interactive whiteboard which has a large square divided into four—with two quarters shaded in—and the question written beside it]. The two halves, Alex, if you colored in that and that [the two quarters in the unshaded half] it would be like a whole [meaning all four quarters would then be shaded which is the same as a whole], but if you just color in that [the part that's already shaded], that's half.	
	Alex: So how does that make the half bigger?	Alex resists the idea that one half and two quarters are equal twice. (QU)
	Chloe: No . . .	
	Alex: You're saying that the half is bigger.	
Chloe disagrees twice in succession. (RESP-D)	Chloe: No, it's exactly the same amount.	
	Alex: [pauses . . . thinking . . . a grin] What's the question again?	This is a critical moment where Alex is revisiting the question herself. (RESP-C) (QU)

(Continued on next page)

TABLE 4a
Segment 3: Chloe Tries Other Ways to Explain Her Thinking After the Teacher Prompts Her to Explain (3 minutes, 20 seconds excerpt) *(Continued)*

Chloe		Alex
Chloe uses a fourth form of explanation by introducing hypothetical actions by the teacher. (EXP)	Chloe: . . . If the teacher came and this was a real item and she cut it between the middle and she divided it like that . . . then she would put it apart [makes two halves] and she would color it like this [shading in one half].	
	Alex: okay, can you repeat that?	(RESP-C)
	Chloe: She'll color in this maybe [the half], and then she'll divide the other one in half.	
	Alex: So she'll just go like that [gesturing to cut the half in half] and split it?	Alex shows how the halves could be further divided in half. (RESP-C) (QU)

TABLE 4b
Summary Analysis of Types of Interaction in Segment 3

	Chloe	Alex	Total
Questioning (QU)	0	5	5
Explaining mathematical thinking (EXP)	5	0	5
Responsibility for learning (RESP)			
Agree (RESP-A)	0	0	0
Disagree (RESP-D)	2	0	2
Seeking clarification (RESP-C)	0	6	6

TABLE 5
Summary of Types of Interaction Across All Segments

	Chloe	Alex	Total
Questioning (QU)	3	5	8
Explaining mathematical thinking (EXP)	8	7	15
Responsibility for learning (RESP)			
Agree (RESP-A)	1	0	1
Disagree (RESP-D)	5	2	7
Seeking clarification (RESP-C)	7	6	13
Total for responsibility			21

two types of interaction fit naturally together in mathematics discussions, even for these young children.

The Role of the Teacher in Small Group Student–Student Interaction

Maggie, the teacher, played an important role in monitoring and scaffolding mathematics discussions, even in the small group setting. On a superficial level, it appeared that the students had no direction because the teacher did not engage with the students except on one brief occasion. However, on closer inspection of the context, we can see that Maggie did a number of important things to support the pairs math-talk that are in strong alignment with Empson's (2003) factors of success for low-performing students: (a) She gave the students a challenging math task that built on prior learning in the lesson sequence. (b) She provided structure for student math-talk with the use of the math-talk guidelines and the popsicle stick sentence starters. (c) She legitimized both students' thinking by allowing them to continue with their discussion without intervening. (d) She encouraged Chloe to explain her thinking, instilling further confidence and a sense of voice.

CONCLUSION

In this article, we have illustrated a sustained discussion between two low-achieving female Grade 1 students. With appropriate norms established and scaffolding materials in place, these students engaged in a productive student–student mathematical discussion that increased their understanding of equivalent fractions, even at a young age and even when discussions appeared to lead the students astray in their mathematics thinking (Moss & Case, 1999). This concurs with Yackel and Cobb's (1996) findings that when teachers establish productive norms for math-talk, student thinking is extended and deepened. Through discussion, a tentative Chloe was able to grab onto emerging but important conceptual ideas about fractions and begin to voice them using mathematical language, and Alex eventually became aware of flaws in her thinking and began to ask critical questions of her partner, resulting in a clearer understanding over several days of investigation.

The Hufferd-Ackles et al. (2004) framework, although designed to describe whole-class interactions, offered us insights into the types of interactions, the frequency of these interactions, and the shifts in these interactions through the progression of the pairs math-talk. In our study we eliminated the category "source of mathematics ideas" because the students were the sole human source of mathematics ideas, and we amplified the "responsibility" category to include subcategories to describe the subtleties within the category. Further testing of a more refined version of this tool explicitly designed for student–student pairs interaction provides a next step for research.

The results of this study have implications for the professional development efforts of teachers, and related implications regarding student coconstruction of foundational mathematics concepts. We have learned that letting student ideas "stand longer" in sustained student–student math-talk requires scaffolding such as the use of math-talk guidelines, carefully selected math problems, and task-specific manipulatives. With these supports, but minimal teacher redirection, low-achieving students can coconstruct foundational mathematics concepts in ways that are empowering for the

students. In this study, we have found that lesson study is a learning model that provides teachers and researchers with the opportunity to look closely at student work and student thinking. This focused attention on the student at the center of the learning is the very foundation of lesson study activity and at the heart of our research program. In Maggie's case, she used video episodes to look at and listen even more closely to her students' thinking. We have observed that teachers who are engaged in lesson study adopt a learning stance that encourages them to dig deeper into student learning and, rather than micromanaging the learning situation, to step back in order to allow students to do the mathematics thinking.

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